

Neutron negative charge density: Exclusive-Inclusive Connection

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Outline-

1. How **not to** and how **to** analyze electromagnetic form factors- transverse density
2. Model independent proton, neutron transverse charge density
3. Interpret neutron charge density -relate to high x pdfs

Transverse Charge Densities.

[Gerald A. Miller](#), arXiv:1002.0355 [nucl-th] ARNPS

Interpretation of Sachs - $G_E(Q^2)$ is Fourier transform of
charge density

$$R^2 = -6 \frac{dG_E(Q^2)}{dQ^2} \Big|_{Q^2=0}$$

**Correct non-relativistic:
wave function invariant under Galilean
transformation, BUT IT IS WRONG**

**Relativistic : wave function is frame
dependent, initial and final states differ**

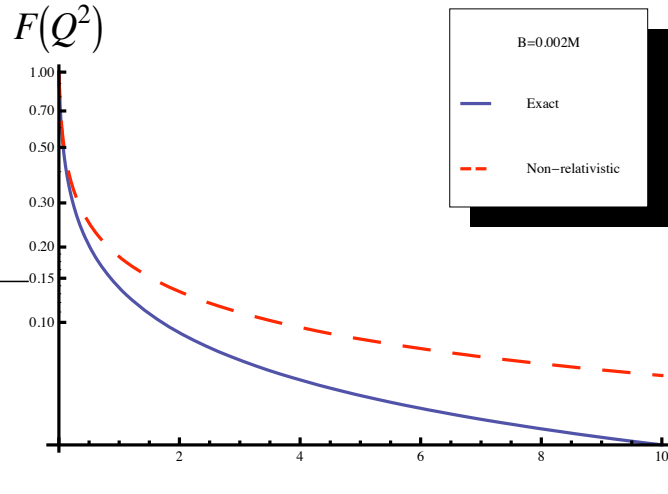
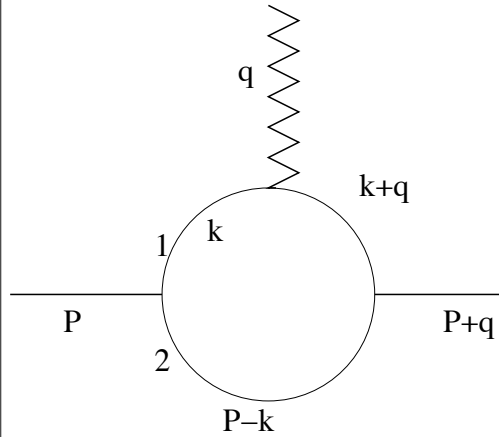
interpretation of Sachs FF is wrong

Final wave function is boosted from initial

Need relativistic treatment

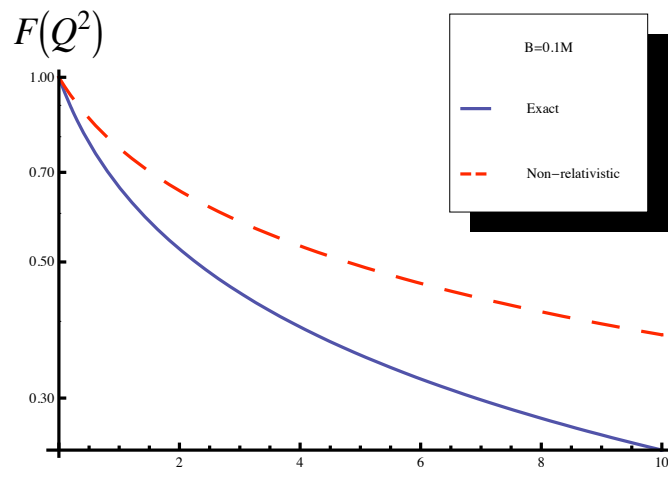
Toy model

GAM, Phys.Rev.C80:045210,2009.
 Scalar meson M , made of two scalar mesons, m
 IF $(M-2m)/M$, small non-relativistic works



deuteron kinematics are non-relativistic: extract neutron structure function should be possible

$(M-2m)/M=0.002$



Relativity needed

$(M-2m)/M=0.1$

Exact vs non-relativistic Form factors for the case $m_1 = m_2 = m$.

Light front, Infinite momentum frame

“Time”, $x^+ = x^0 + x^3$, “Evolve”, $p^- = p^0 - p^3$

“Space”, $x^- = x^0 - x^3$, “Momentum”, p^+ (Bjorken)

Transverse position, momentum \mathbf{b}, \mathbf{p}

These variables are used in GPDs, TMDs, standard variables

transverse boosts in kinematic subgroup

$$\mathbf{k} \rightarrow \mathbf{k} - k^+ \mathbf{v}$$

space – like $q^\mu, q^+ = 0,$

momentum transfer in transverse direction

**then density is 2 Dimensional
Fourier Transform**

Model independent transverse charge density

$$J^+(x^-, \mathbf{b}) = \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) \quad \text{Charge Density}$$

$$\rho_\infty(x^-, \mathbf{b}) = \langle p^+, \mathbf{R} = \mathbf{0}, \lambda | \sum_q e_q q_+^\dagger(x^-, b) q_+(x^-, b) | p^+, \mathbf{R} = \mathbf{0}, \lambda \rangle$$

$$F_1 = \langle p^+, \mathbf{p}', \lambda | J^+(0) | p^+, \mathbf{p}, \lambda \rangle$$

$$\rho(b) \equiv \int dx^- \rho_\infty(x^-, \mathbf{b}) = \int \frac{Q dQ}{2\pi} F_1(Q^2) J_0(Qb)$$

Density is $u - \bar{u}$, $d - \bar{d}$

Soper '77

Impact parameter dependent GPD Burkardt

Probability that quark at b from CTM has long momentum fraction x

0 skewness, $\xi = 0$

$$\rho^q(b, x) = \int \frac{d^2 q}{(2\pi)^2} e^{-i\mathbf{q}\cdot\mathbf{b}} H_q(x, t = \mathbf{q}^2)$$

$$\rho(b) = \sum_q e_q \int dx \rho^q(b, x)$$

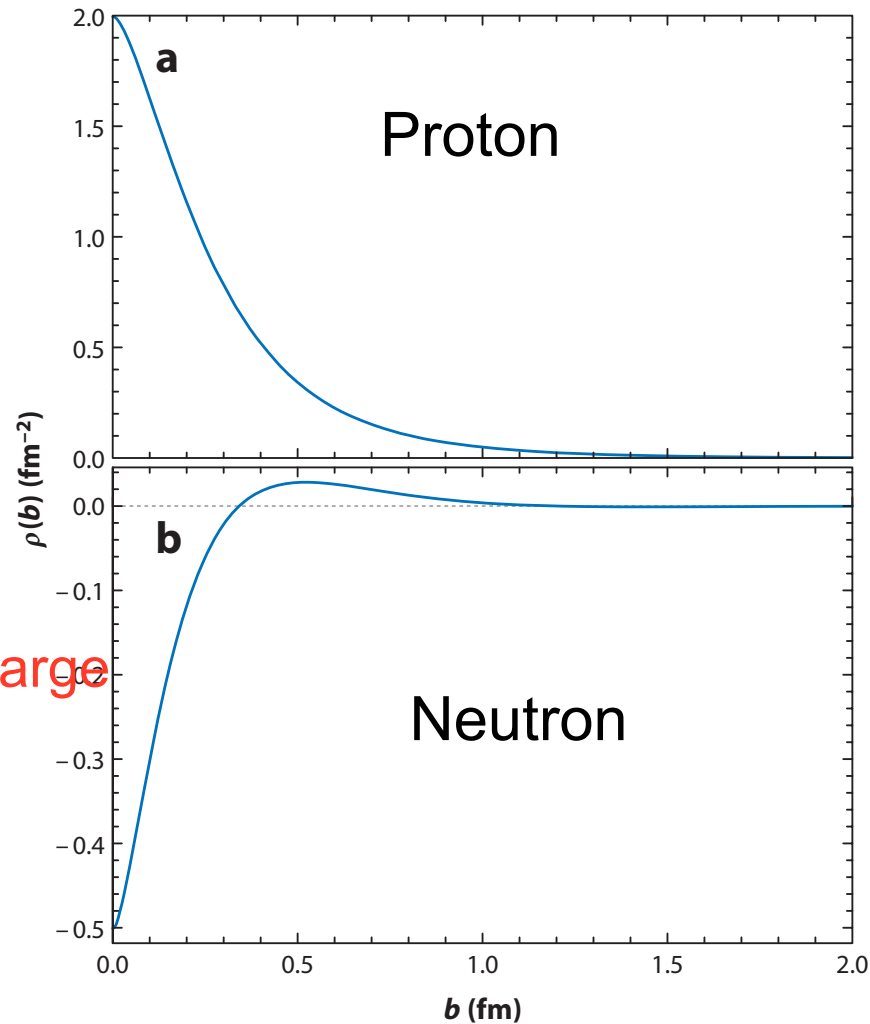
sum rule: integral
of H_q is F

$$\mathbf{R} = \mathbf{0} = \sum_i^N x_i \mathbf{b}_i$$

Quark of $x=1$, must have $b=0$

Transverse density is integral over longitudinal position **or** momenta
example of Parseval's theorem

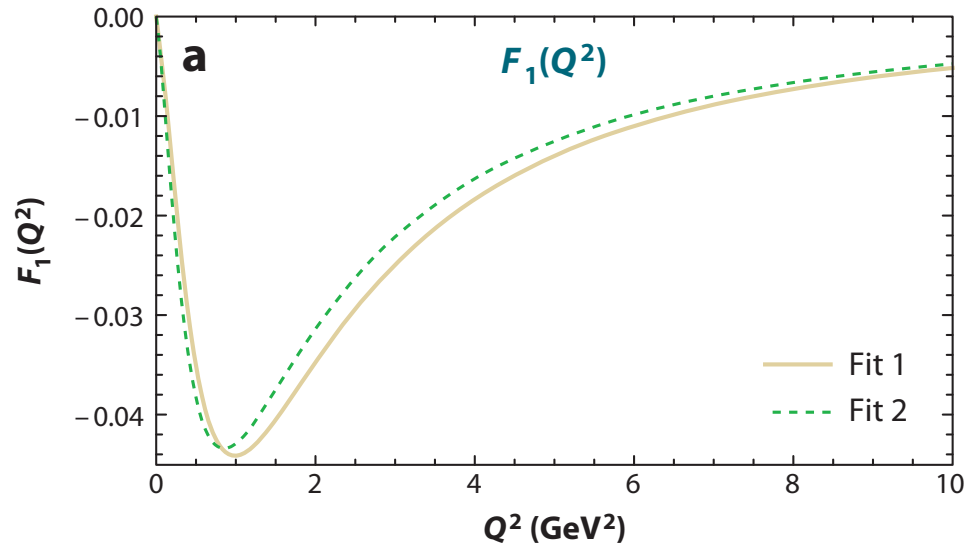
Transverse charge densities from parameterizations (Alberico)



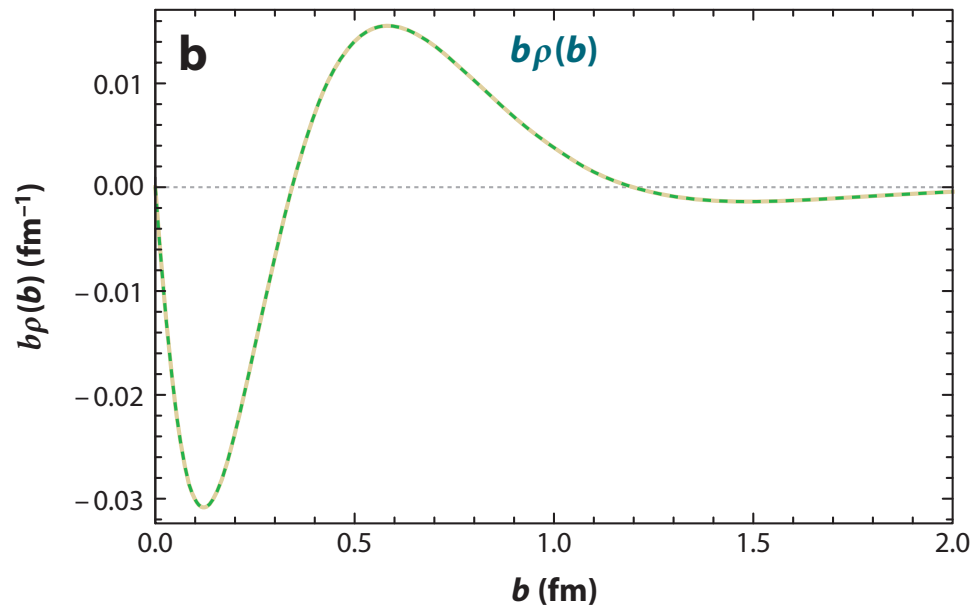
Negative central charge density

Negative central density - GAM PRL '07

Neutron



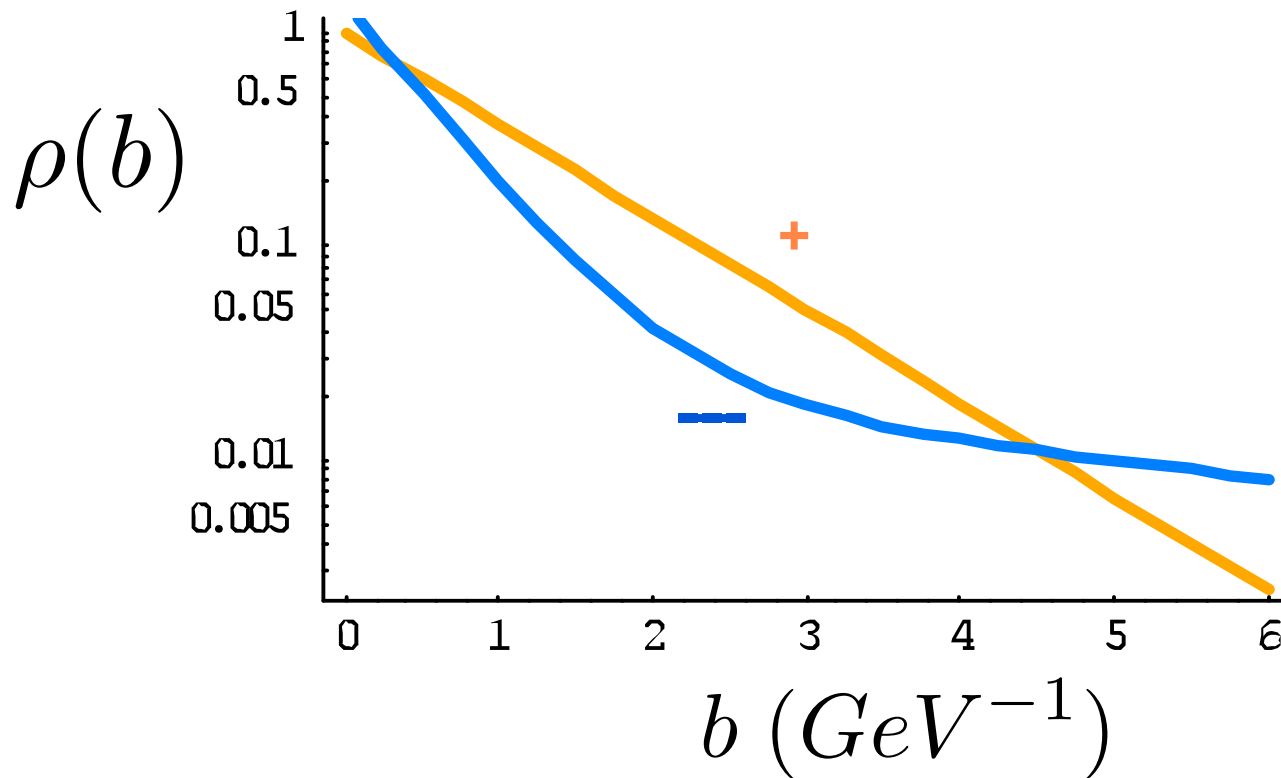
F_1 is
negative, so
is central
density



Negative at
large b , pion
cloud? see
Strikman
Weiss '10

arXiv:1004.3535

Neutron charge density: why?



Neutron interpretation

- Impact parameter gpd Burkardt $\rho(x, b)$
- Drell-Yan-West relation between high x DIS and high Q^2 elastic scattering
- High x related to low b , not uncertainty principle

$$\lim_{x \rightarrow 1} \nu W_2(x) = (1-x)^{2n-1} \leftrightarrow \lim_{Q^2 \rightarrow \infty} F_1(Q^2) \sim \frac{1}{Q^{2n}}, n = 2$$

- Various (reasonable) assumptions needed: power-law wave functions. Is this relationship valid?

$$\pi \text{ DYW} : F(Q^2) \sim \frac{1}{Q^2} \rightarrow \nu W_2 = (1-x)$$

But Reimer data $(1-x)^2$

??????

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Structure functions

- d quarks dominate DIS from neutron at high x
- d quarks dominate at neutron center, or π^-

Density is $u - \bar{u}$, $d - \bar{d}$

π^- is $\bar{u}d$

decreases u contribution

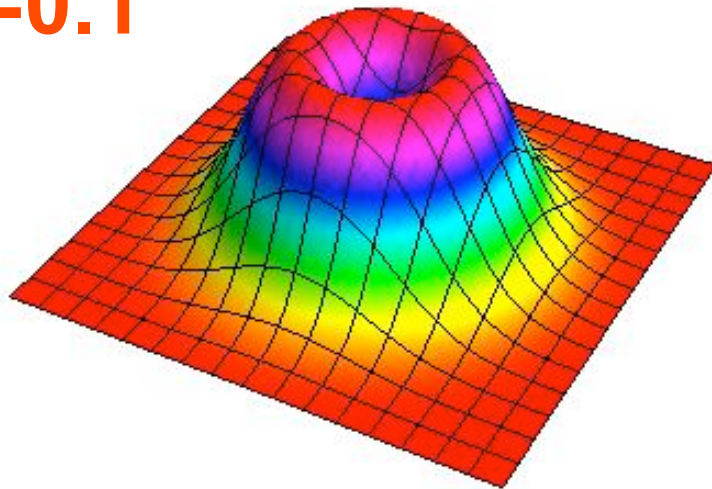
enhances d contribution

Neutron interpretation $\rho(x,b)$

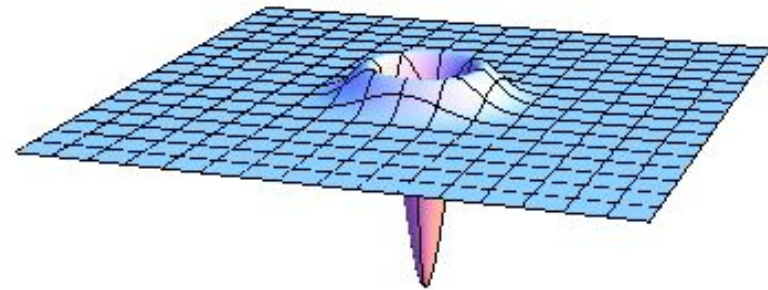
GAM, J. Arrington, PRC78,032201R '08

Guidal et al 05, Diehl et al 05, Ahmad07, Tiburzi04.

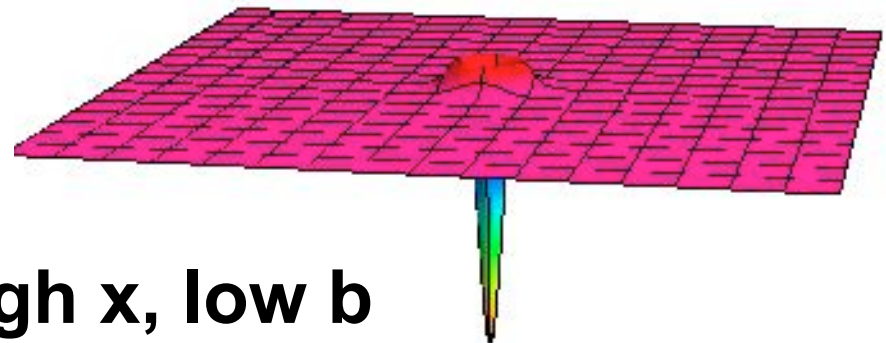
$x=0.1$



$x=0.3$

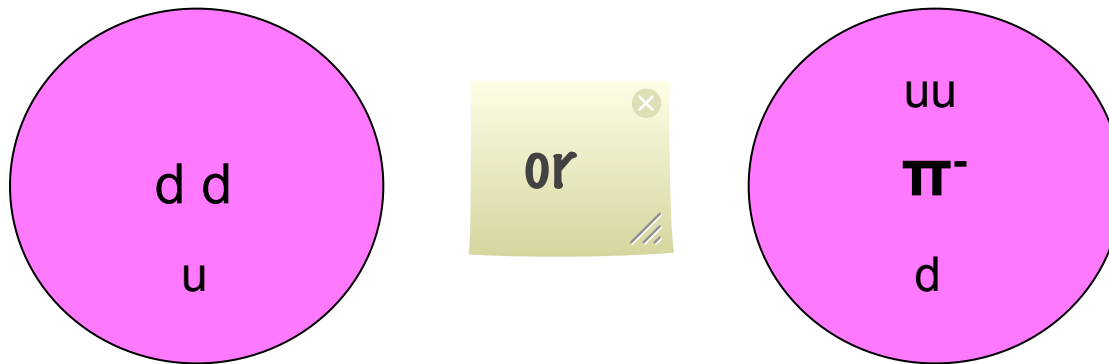


$x=0.5$

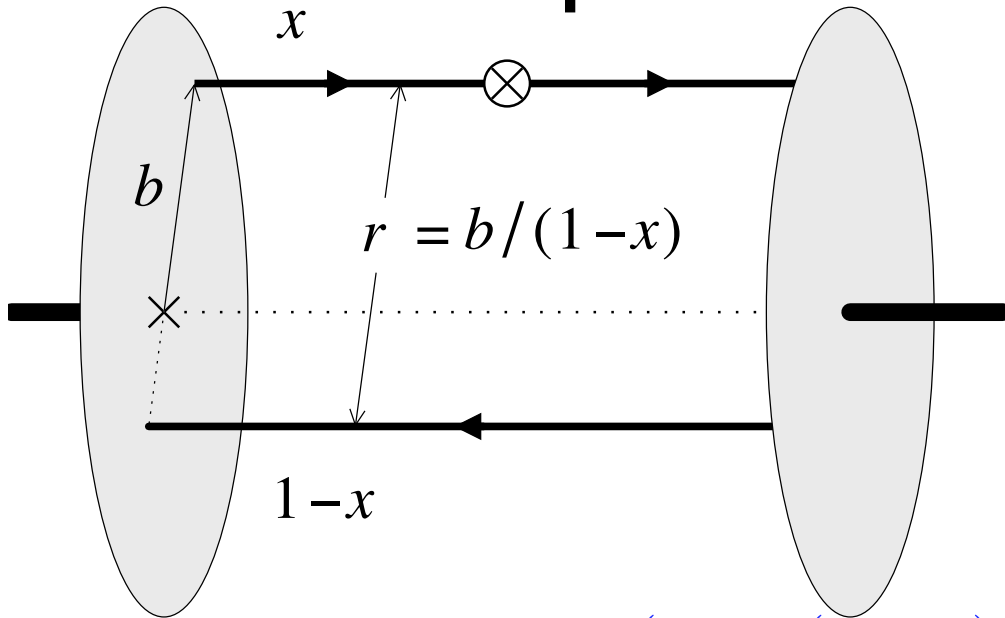


d or π^- dominates at high x, low b

Neutron interpretation



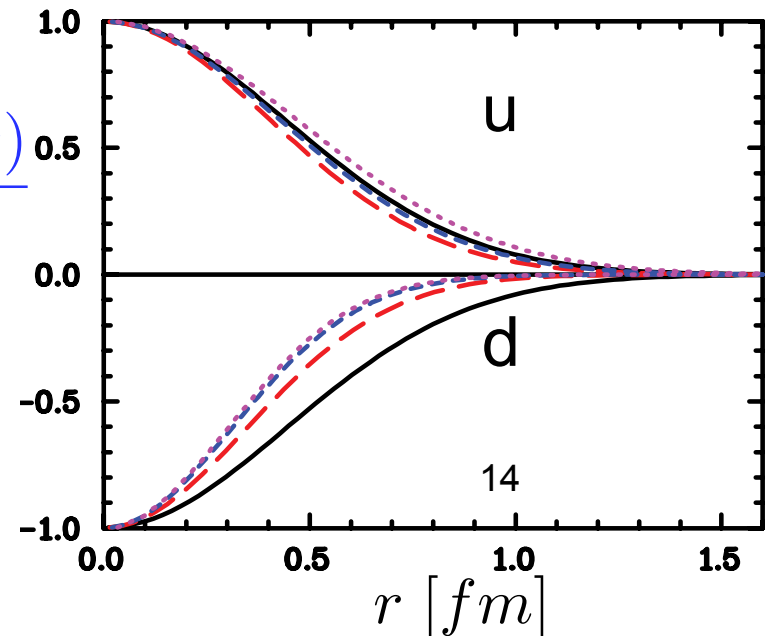
Understand b: quark and spectator system



$$\frac{\rho(b = r(1-x), x)}{\rho(0, x)}$$

Model dependent

Several values of x ,
little variation

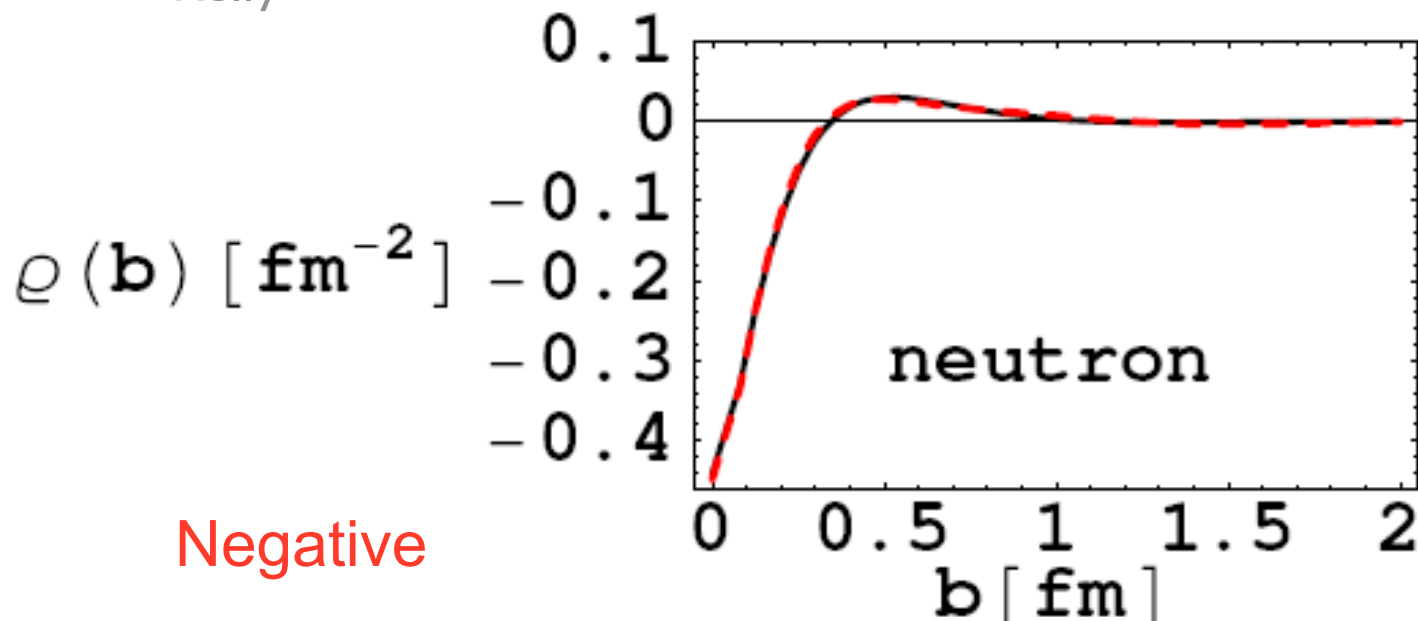
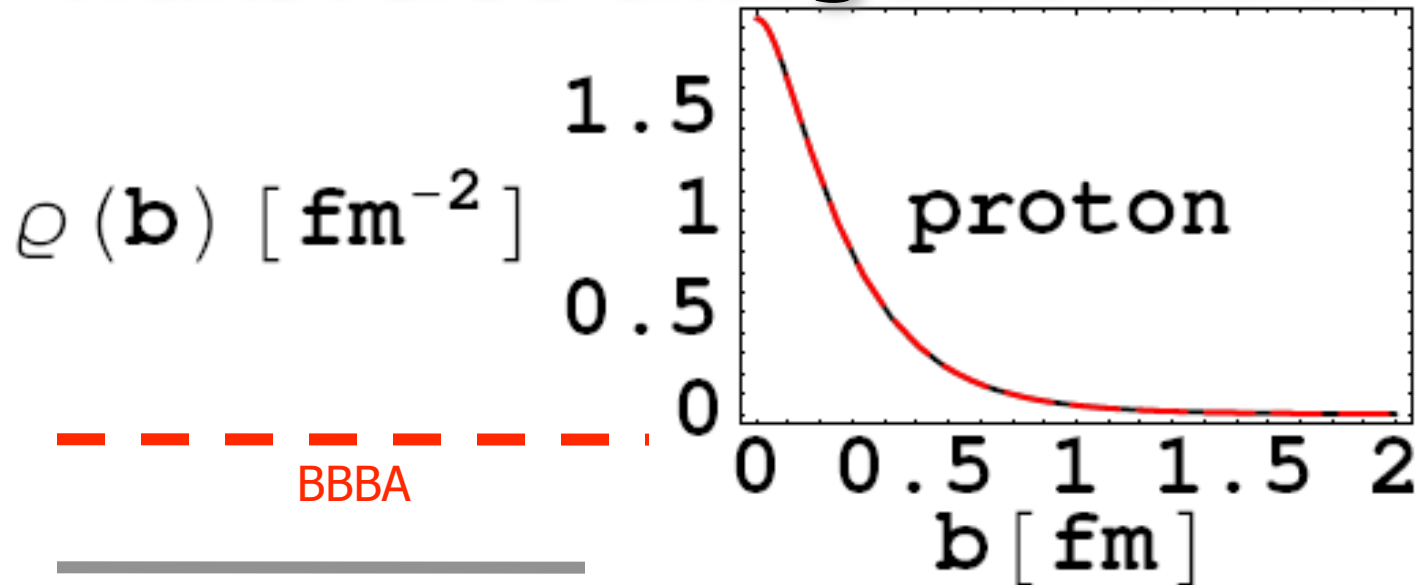


Summary

- **Much data exist, Jlab12 will improve data set**
- **Charge density is not a 3 dimensional Fourier transform of G_E**
- **Interpret form factor as determining transverse charge density**
- **Neutron: Negative central density**
- **Full understanding needs $\rho(b, x)$**
- **Center of neutron: d or π^-**
- **Is Drell-Yan, West relation valid?**

Spares follow

Transverse charge densities



Negative

Relation or **not** between GPD and TMD

GPD :

$$\begin{aligned} & \langle P', S' | \int \frac{dx^-}{4\pi} \bar{q}\left(-\frac{x^-}{2}, \mathbf{0}\right) \gamma^+ q\left(\frac{x^-}{2}, \mathbf{0}\right) e^{ix\bar{p}^+ x^-} | P, S \rangle_{x^+ = 0} \\ &= \frac{1}{2\bar{p}^+} \bar{u}(P', S') \left(\gamma^+ H_q(\xi, t) + i \frac{\sigma^{+\nu} \Delta_\nu}{2M} E_q(x, \xi, t) \right) u(P, S) \end{aligned}$$

TMD :

$$\Phi_q^\Gamma\left(x = \frac{k^+}{P^+}, \mathbf{k}\right) = \langle P, S | \int \frac{d\zeta^- d^2\zeta}{2(2\pi)^3} e^{ik \cdot \zeta} \bar{q}(0) \Gamma q(\zeta) | P, S \rangle_{\zeta^+ = 0}$$

GPD: nucleons have different momenta, but FT local in coordinate space if integrate over x

TMD: nucleons have same momenta, operator is local in momentum space

Both can be obtained Wigner distribution operator

$$W_q^\Gamma(\zeta^-, \zeta, k^+, \mathbf{k}) = \frac{1}{4\pi} \int d\eta^- d^2\eta e^{ik \cdot \eta} \bar{q}(\zeta^- - \frac{\eta^-}{2}, \zeta - \frac{\boldsymbol{\eta}}{2}) \Gamma q(\zeta^- + \frac{\eta^-}{2}, \zeta + \frac{\boldsymbol{\eta}}{2})$$

$$H_q(x, \xi, t) = \langle P', S' | \int \frac{d^2\mathbf{k}}{(2\pi)^2} W_q^{\gamma^+}(\zeta^- = 0, \zeta = 0, k^+, \mathbf{k}) | P, S \rangle$$

$$\Phi_q^\Gamma(x, \mathbf{k}) = \langle P, S | \int \frac{d\zeta^-}{(2\pi)^2} W_q^\Gamma(\zeta^-, \zeta, k^+, \mathbf{k}) | P, S \rangle$$